E

Tensorial Structure of the Inhomogeneous Field Equations (IE)

E.1 Introduction (HE & IE)

The tensorial structure of the IE is derived from the Bianchi identity of differential geometry

$$D \wedge T^a = R^a{}_b \wedge q^b \tag{E.1}$$

i.e.

$$d \wedge T^a = R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b \tag{E.2}$$

or

$$d \wedge T^a = -q^b \wedge R^a{}_b - \omega^a{}_b \wedge T^b \tag{E.3}$$

Restoring the indices of the base manifold in eqn. (E.3)

$$\boxed{d \wedge T^a{}_{\mu\nu} = -\left(q^b \wedge R^a{}_{b\mu\nu} + \omega^a{}_b \wedge T^b{}_{\mu\nu}\right)} \tag{E.4}$$

Eqn. (E.4) becomes the homogeneous field equations (HE) using the rules:

$$A^a{}_\mu = A^{(0)} q^a{}_\mu \tag{E.5}$$

$$F^a{}_{\mu\nu} = A^{(0)} T^a{}_{\mu\nu} \tag{E.6}$$

Therefore the HE is:

$$\boxed{\begin{aligned} d \wedge F^a{}_{\mu\nu} &= -A^{(0)}(q^b \wedge R^a{}_{b\mu\nu} + \omega^a{}_b \wedge T^a{}_{\mu\nu}) \\ &\sim 0 \,(\text{experimentally}) \end{aligned}} \tag{E.7}$$

The IE is obtained from eqn (E.7) by taking the appropriate Hodge duals:

$$\tilde{F}^a{}_{\rho\sigma} = \frac{1}{2} |g|^{\frac{1}{2}} \epsilon_{\rho\sigma}{}^{\mu\nu} F^a{}_{\mu\nu} \tag{E.8}$$

$$\tilde{R}^a{}_{b\rho\sigma} = \frac{1}{2}|g|^{\frac{1}{2}}\epsilon_{\rho\sigma}{}^{\mu\nu} R^a{}_{b\mu\nu} \tag{E.9}$$

in the general 4-D manifold.
We therefore multiply both sides of eqn (E.7) by:

$$\frac{1}{2}|g|\epsilon_{\rho\sigma}{}^{\mu\nu}$$

to obtain the tensorial representation of the IE:

$$d \wedge \tilde{F}^a{}_{\mu\nu} = -A^{(0)}(q^b \wedge \tilde{R}^a{}_{b\mu\nu} + \omega^a{}_b \wedge \tilde{T}^b{}_{\mu\nu}) \tag{E.10}$$

The charge-current density of field theory is therefore:

$$J^a = -\frac{A^{(0)}}{\mu_0}\left(q^b \wedge \tilde{R}^a{}_b + \omega^a{}_b \wedge \tilde{T}^b\right) \tag{E.11}$$

and is a vector valued three-form of differential geometry, defined by:

$$\boxed{d \wedge \tilde{F}^a = \mu_0 J^a} \tag{E.12}$$

Eqn. (E.12) is the same equation as:

$$\boxed{\begin{aligned}\partial_\mu F^{a\mu\nu} &= -A^{(0)}(q^b{}_\mu R^a{}_b{}^{\mu\nu} + \omega^a{}_{b\mu} T^{b\mu\nu}) \\ &= \mu_0 J^{a\nu}\end{aligned}} \tag{E.13}$$

E.2 The Coulomb Law ($\nu = 0$)

The Coulomb Law in the unified field theory is given by $\nu = 0$, and is:

$$\partial_1 F^{a10} + \partial_2 F^{a20} + \partial_3 F^{a30} = \mu_0 J^{a0} \tag{E.14}$$

where:

$$\begin{aligned}J^{a0} = -\frac{A^{(0)}}{\mu_0}&\left(q^b{}_1 R^a{}_b{}^{10} + q^b{}_2 R^a{}_b{}^{20} + q^b{}_3 R^a{}_b{}^{30}\right. \\ &\left.+\omega^a{}_{b1} T^{b10} + \omega^a{}_{b2} T^{b20} + \omega^a{}_{b3} T^{b30}\right)\end{aligned} \tag{E.15}$$

i.e.

$$\boxed{\begin{aligned}\nabla \cdot \boldsymbol{E}^a = -\,\phi^{(0)}&\left(q^b{}_1 R^a{}_b{}^{10} + q^b{}_2 R^a{}_b{}^{20} + q^b{}_3 R^a{}_b{}^{30}\right. \\ &\left.+\omega^a{}_{b1} T^{b10} + \omega^a{}_{b2} T^{b20} + \omega^a{}_{b3} T^{b30}\right)\end{aligned}} \tag{E.16}$$

E.2.1 Notes on the Coulomb Law

It can be seen from the familar vector notation in eqn. (E.16) that the origin of the Coulomb Law is spacetime. Using the constitutive equations:

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b \tag{E.17}$$

$$R^a{}_b = d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b \tag{E.18}$$

the right hand side of eqn (E.16) becomes a function only of the tetrad and the spin connection. The Riemann form must always obey the second Bianchi identity.

$$D \wedge R^a{}_b = 0 \tag{E.19}$$

Therefore eqn (E.17) to (E.19) are constraints on the variables on the right hand side of eqn. (E.16). Einstein field theory of gravitation is given by the limit:

$$T^a = 0 \tag{E.20}$$

$$D \wedge T^a = R^a{}_b \wedge q^b = 0 \tag{E.21}$$

and Newtonian gravitation is the weak field limit of eqn (E.20) and (E.21). In the Einstein and Newton theories of pure gravitation therefore:

$$q^b{}_1 R^a{}_b{}^{10} + q^b{}_2 R^a{}_b{}^{20} + q^b{}_3 R^a{}_b{}^{30} \neq 0 \tag{E.22}$$

The electromagnetic field is torsion of spacetime (eqn.(E.6)) so from eqn. (E.20) there is no electro-magnetism in the Einstein or Newton theories of gravitation. This is of course a self-consistent result. However, it is seen further from eqns (E.16) and (E.22) that Einstein or Newtonian gravitation does not affect the Coulumb Law.

In order for gravitational forces to change the Coulomb Law the condition is:

$$\boxed{q^b{}_1 R^a{}_b{}^{10} + q^b{}_2 R^a{}_b{}^{20} + q^b{}_3 R^a{}_b{}^{30} \neq 0} \tag{E.23}$$

this condition is compatible with:

$$R^a{}_b \wedge q^b = 0 \tag{E.24}$$

and to:

$$R_{\mu\nu\rho\sigma} + R_{\rho\mu\nu\sigma} + R_{\nu\rho\mu\sigma} = 0 \tag{E.25}$$

Eqn. (E.25) means that:

$$\Gamma^\kappa{}_{\mu\nu} = \Gamma^\kappa{}_{\nu\mu} \tag{E.26}$$

where $\Gamma^\kappa_{\mu\nu}$ is the Christoffel symbol.

Eqn (E.26) means that the gravitational torsion tensor vanishes:

$$T^\kappa{}_{\mu\nu} = q^\kappa{}_\mu T^a{}_{\mu\nu} = \Gamma^\kappa{}_{\mu\nu} - \Gamma^\kappa{}_{\nu\mu} = 0. \tag{E.27}$$

E.3 The Ampère Maxwell Law ($\nu = 1,2,3$)

The Ampère Maxwell Law is:

$$\partial_0 F^{a01} + \partial_2 F^{a21} + \partial_3 F^{a31} = \mu_0 J^{a1}\ (\nu = 1) \tag{E.28}$$
$$\partial_0 F^{a02} + \partial_1 F^{a12} + \partial_3 F^{a32} = \mu_0 J^{a2}\ (\nu = 2) \tag{E.29}$$
$$\partial_0 F^{a03} + \partial_1 F^{a13} + \partial_2 F^{a23} = \mu_0 J^{a3}\ (\nu = 3) \tag{E.30}$$

where:

$$\begin{aligned} J^{a1} = -\frac{A^{(0)}}{\mu_0} \big(& q^b{}_0 R^a{}_b{}^{01} + q^b{}_2 R^a{}_b{}^{21} + q^b{}_3 R^a{}_b{}^{31} \\ & + \omega^a{}_{b0} T^{b01} + \omega^a{}_{b2} T^{b21} + \omega^a{}_{b3} T^{b31} \big) \end{aligned} \tag{E.31}$$

$$\begin{aligned} J^{a2} = -\frac{A^{(0)}}{\mu_0} \big(& q^b{}_0 R^a{}_b{}^{02} + q^b{}_1 R^a{}_b{}^{12} + q^b{}_3 R^a{}_b{}^{32} \\ & + \omega^a{}_{b0} T^{b02} + \omega^a{}_{b1} T^{b12} + \omega^a{}_{b3} T^{b32} \big) \end{aligned} \tag{E.32}$$

$$\begin{aligned} J^{a3} = -\frac{A^{(0)}}{\mu_0} \big(& q^b{}_0 R^a{}_b{}^{03} + q^b{}_1 R^a{}_b{}^{13} + q^b{}_2 R^a{}_b{}^{23} \\ & + \omega^a{}_{b0} T^{b03} + \omega^a{}_{b1} T^{b13} + \omega^a{}_{b2} T^{b23} \big) \end{aligned} \tag{E.33}$$

E.3.1 Notes on the Ampère Maxwell Law

It is seen from eqns (E.31) to (E.33) that electric current is spacetime. The same consideration apply as to the Coulomb Law, because both laws are part of the IE. Therefore gravitational torsion is needed to generate electric current and electric power.